Name_____ Date_____ Period_____

Worksheet 3.1—Extrema on an Interval

Show all work. No calculator, except on problem #5d & 6.

Short Answer

1. Let f be the functioned defined on $[-1, 2]$ by $f(x) = 3x^{2/3} - 2x$.
    (a) Find the domain of $f(x)$.

    (b) Find $f'(x)$ and the domain of $f'(x)$.

    (c) Determine if the EVT applies to $f(x)$ on the given interval. If it does, find the max and min values
        guaranteed by the theorem. Justify using the Closed Interval Argument. Show the work that leads to
        your answer.

2. Sketch the graph of a function f that is continuous on $[1, 5]$ and has an absolute minimum at $x = 1$, an
    absolute maximum at $x = 5$, a local maximum at $x = 2$, and a local minimum at $x = 4$. Answers will
    vary.

3. Sketch the graph of f and use your sketch to find the absolute and local extrema of f on the indicated domains.

(a) $f(x) = \begin{cases} 1-x, & 0 \le x < 2 \\ 2x-4, & 2 \le x \le 3 \end{cases}$

(b) $f(x) = \begin{cases} x^2, & -1 \le x < 0 \\ 2-x^2, & 0 \le x \le 1 \end{cases}$

4. Find the critical values of the functions over their domains. **Remember that a critical value MUST be in the domain of the function, though it may not be in the domain of that function's derivative function. Also, be on CUSP ALERT!**

(a) $x(t) = 3t^4 + 4t^3 - 6t^2$

(b) $f(z) = \dfrac{z+1}{z^2 + z + 1}$

(c) $g(t) = 5t^{2/3} + t^{5/3}$

(d) $g(t) = \sqrt{t}(1-t)$

(e) $g(\theta) = 4\theta - \tan\theta$

(f) $f(x) = x \ln x$

(g) $G(x) = \sqrt[3]{x^2 - x}$

(h) $f(x) = xe^{2x}$

(i) $g(x) = |2x+3|$

5. Find the absolute extrema of f on the given interval.

   (a) $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2,3]$ (b) $f(x) = (x^2 - 1)^3$, $[-1,2]$

   (c) $f(t) = \sqrt[3]{t}(8 - t)$, $[0,8]$ (d) (Calculator permitted at end)

   $$f(x) = \sin x + \cos x, \; \left[0, \frac{\pi}{3}\right]$$

6. (Calculator Permitted) Using your calculator's equation solving capability (not just its max/min finding ablity), find the extrema of the $f(x) = \dfrac{\ln x - e^x}{x}$ on the interval $[1,3]$. Be sure to show the equation you're solving and you justification via the Closed Interval Argument.

7. Prove that the function $f(x) = x^{101} + x^{51} + x + 1$ has neither a local maximum nor a local minimum by analyzing the continuity and the sign of $f'(x)$, the derivative function.

Multiple Choice

_____ 8. Find all the critical values, $x = c$, of the function $g(x) = 5x + \sin 5x$ in $(0, \infty)$, where $n = 0, 1, 2, \ldots$.

(A) $c = \dfrac{3\pi}{5}n + \dfrac{\pi}{5}$ (B) $c = \dfrac{\pi}{5}n$ (C) $c = \dfrac{2\pi}{5}n + \dfrac{\pi}{5}$ (D) $c = \dfrac{4\pi}{5}n + \dfrac{\pi}{5}$ (E) $c = \dfrac{\pi}{5}n + \dfrac{\pi}{5}$

_____ 9. Determine the absolute maximum value of $f(x) = \dfrac{5 + 2x}{x^2 + 14}$ on the interval $[-2, 4]$.

(A) $\dfrac{1}{18}$ (B) $\dfrac{13}{30}$ (C) $\dfrac{8}{7}$ (D) $\dfrac{1}{2}$ (E) None

_____ 10. Find all the critical values of f when $f(x) = x^{4/5}(x-5)^2$. Be sure to factor out least powers after differentiating.

(A) $0, \dfrac{5}{7}$　　(B) $\dfrac{10}{7}, 5$　　(C) $\dfrac{5}{7}, 5$　　(D) $0, \dfrac{5}{7}, 5$　　(E) $0, \dfrac{10}{7}$　　(F) $0, \dfrac{10}{7}, 5$

11. Let f be the function defined by $f(x) = x\sqrt{1-x^2} + 2$ on $[-1, 0]$.

_____ (i) Find $f'(x)$ by factoring out least powers.

(A) $f'(x) = \dfrac{1-2x^2}{\sqrt{1-x^2}}$

(B) $f'(x) = \dfrac{\sqrt{1-x^2}}{2x^2}$

(C) $f'(x) = \dfrac{2-x^2}{\sqrt{1-x^2}}$

(D) $f'(x) = \sqrt{1-x^2}$

(E) $f'(x) = \dfrac{2x}{\sqrt{1-x^2}}$

_____ (ii) Find all the critical values of $f(x)$ in $(-1, 0)$.

(A) $x = -\dfrac{1}{\sqrt{2}}$

(B) $x = \pm\dfrac{1}{2}$

(C) $x = \pm\dfrac{1}{\sqrt{2}}$

(D) $x = \dfrac{1}{2}$

(E) $x = \dfrac{1}{\sqrt{2}}$

_____ (iii) Determine the minimum value of $f(x)$ on $[-1, 0]$.

(A) $\dfrac{1}{\sqrt{2}}$

(B) $\dfrac{5}{2}$

(C) 1

(D) $\dfrac{3}{2}$

(E) $\dfrac{1}{2}$

12. Let f be the function defined by $f(x) = \sin x - \cos^2 x$ on $[0, 2\pi]$.

_____ (i) Find $f'(x)$.

(A) $f'(x) = \sin x(1 + 2\cos x)$
(B) $f'(x) = \cos x(1 + 2\sin x)$
(C) $f'(x) = -\sin x(1 + 2\cos x)$
(D) $f'(x) = \cos x(1 - 2\sin x)$
(E) $f'(x) = -\cos x(1 + 2\sin x)$

_____ (ii) Find al' the critical values of $f(x)$ in $(0, 2\pi)$.

(A) $\dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}$

(B) $\dfrac{\pi}{2}, \dfrac{4\pi}{3}, \dfrac{3\pi}{2}, \dfrac{5\pi}{3}$

(C) $\dfrac{\pi}{2}, \dfrac{7\pi}{6}, \dfrac{3\pi}{2}, \dfrac{11\pi}{6}$

(D) $\dfrac{\pi}{2}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{3\pi}{2}$

(E) $\dfrac{2\pi}{3}, \pi, \dfrac{4\pi}{3}$

_____ (iii) Determine the absolute maximum value of $f(x)$ on $[0, 2\pi]$.

(A) -1

(B) $-\dfrac{5}{4}$

(C) $\dfrac{5}{4}$

(D) 1

(E) $-\dfrac{3}{4}$

_____ 13. Let f be the function defined by $f(x)=2\left(x-\dfrac{1}{16x^2}\right)$, $x \neq 0$. Determine the absolute maximum value of f on $(-\infty, -1]$. Hint: find the domain of f, the critical values of f, then look at the sign of f' for all values in the specified interval.

(A) $-\dfrac{17}{8}$

(B) No max value

(C) $-\dfrac{3}{2}$

(D) $\dfrac{3}{2}$

(E) $\dfrac{17}{8}$

_____ 14. An advertisement is run to stimulate the sale of cars. After t days, $1 \leq t \leq 48$, the number of cars sold is given by $N(t)=4000+45t^2-t^3$. On what day does the maximum **rate of growth** of sales occur, that is, on what day is $N'(t)$ a maximum on the given interval?
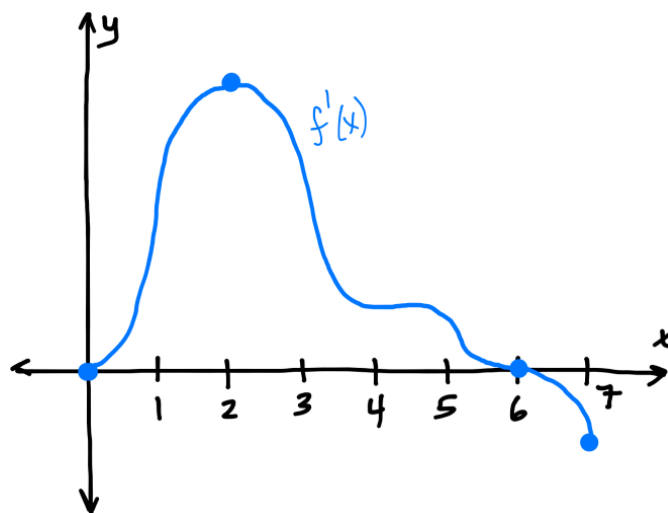
(A) day 17

(B) day 13

(C) day 15

(D) day 16

(E) day 14

_____ 15. The graph above is the graph of $f'(x)$, the derivative of some function $f(x)$. Use the graph above to determine the x-value at which the function $f(x)$ achieves its absolute maximum.

(A) 0   (B) 2   (C) 4   (D) 6   (E) 7

(A) 0   (B) 2   (C) 4   (D) 6   (E) 7